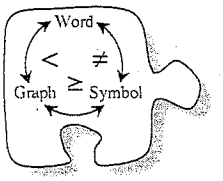
JUDO MATH

BLUE BELT TRAINING

INEQUALITIES

ALGEBRA DISCIPLINE

6.1.1 How do these compare?

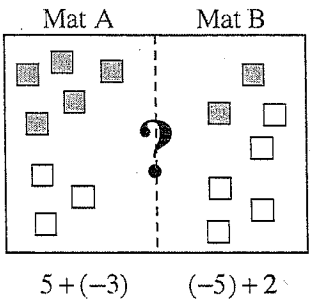


Comparing Expressions

In Chapter 4, you worked with writing and simplifying expressions. As you wrote expressions, you learned that it was helpful to simplify them by combining like terms and removing zeros. In this lesson, you and your teammates will use a tool for comparing expressions. The tool will allow you to determine whether one expression is greater than the other or if they are equivalent ways of writing the same thing (that is, if they are equal).

Remember that to represent expressions with algebra tiles, you will need to be very careful about how positives and negatives are distinguished. To help you understand the diagrams in the text, the legend at right will be placed on every page containing a mat. It shows the shading for +1 and –1. This model also represents a zero pair.

6-1. COMPARING EXPRESSIONS



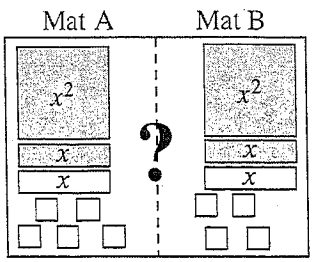
Ignacio and Oliver were playing a game. Each of them grabbed a handful of algebra tiles. They wanted to see whose expression had the greater value.

Two expressions can be compared by dividing the expression mat in half to change it into an **Expression Comparison Mat**. Then the two expressions can be built side by side and compared to see which one is greater.

- Oliver put his tiles on Mat A in the picture above and described it as $5+(-3)$.
- Ignacio put his tiles on Mat B and said it was $(-5)+2$.

With your team, find two different methods to simplify the two expressions so you can compare them. Which side of the mat is larger?

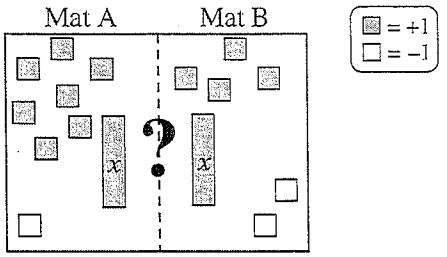
6-2. Using your Expression Comparison Mat, build the two expressions at right. Find a way to determine which side is greater, if possible. Show your work by sketching it on the Lesson 6.1.1B Resource Page. Be ready to share your conclusion and your justification.



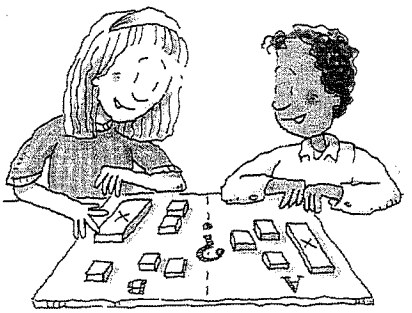
6-3. MORE COMPARING EXPRESSIONS – Is one expression greater?

Consider how you were able to compare the expressions in the previous problems. When is it possible to remove tiles to compare the expressions on the mats? In this problem, you will work with your team to identify two different "legal moves" for simplifying expressions.

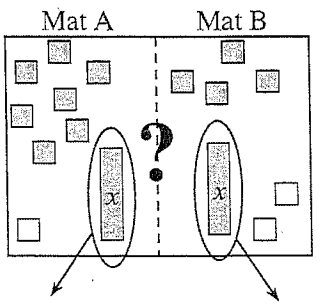
Build the mat below using tiles and simplify the expressions. Record your work by drawing circles around the zeros or the balanced sets of tiles that you remove in each step on the Lesson 6.1.1B Resource Page. Which expression is greater?



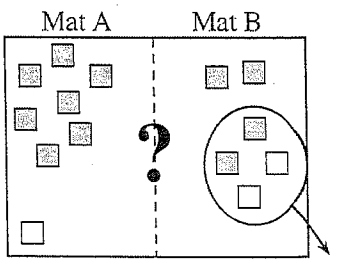
6-4. There are two kinds of moves you could use in problem 6-3 to simplify expressions with algebra tiles. First, you could remove zeros. Second, you could remove matching (or balanced) sets of tiles from both sides of the mat. Both moves are shown in the figures below. Justify why each of these moves can be used to simplify expressions.



Removing Balanced Sets



Removing Zeros



6-5. WHICH SIDE IS GREATER?

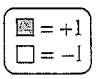
Expression Comparison Mat

For each of the problems below, use the Lesson 6.1.1 Resource Page and:

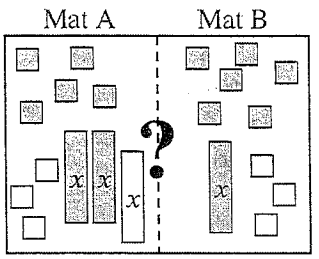
- Build the two expressions on your mat.
- Write an expression for each side below the mats for parts (a) through (d) OR draw the tiles in the space given on the resource page for parts (e) and (f).

below

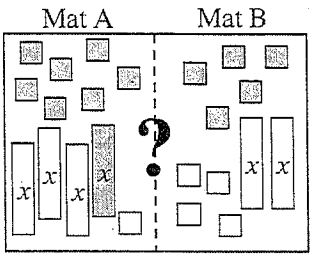
- Use legal moves to determine which mat is greater, if possible. Record your work by drawing circles around the zeros or the balanced (matching) sets of tiles that you remove in each problem.



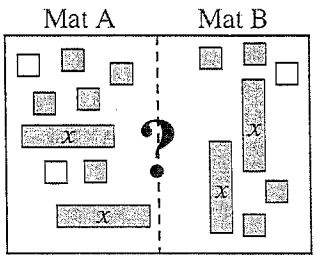
a.



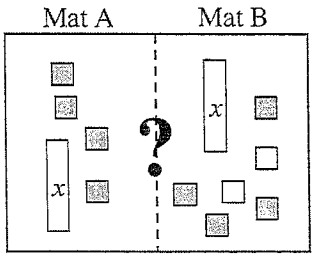
b.



c.



d.

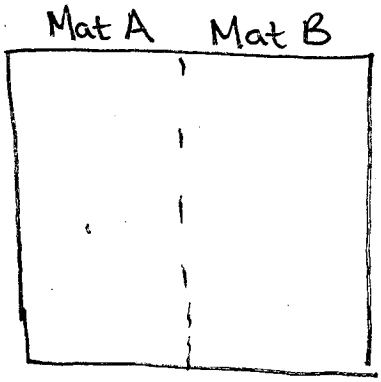


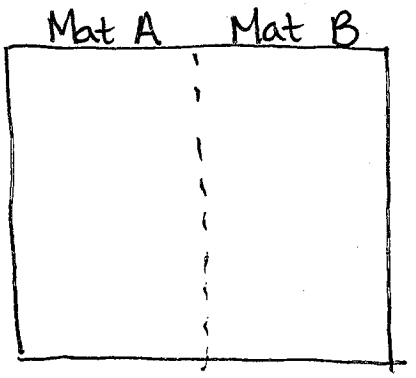
e. Mat A: $3x-4-2$

Mat B: $3(x-1)$

f. Mat A: $5+(-3x)+5x$

Mat B: $x^2+2x+1-x^2$





(3)

READ!! ↓ ↓

MATH NOTES

METHODS AND MEANINGS

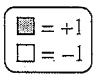
Inequality Symbols

Just as the symbol "=" is used in mathematics to represent that two quantities are equal, the **inequality symbols** at right are used to describe the relationships between quantities that are not necessarily equal. Examples: $3<7$, $14\leq 14$, $-7<-3$, $19\geq 14$.

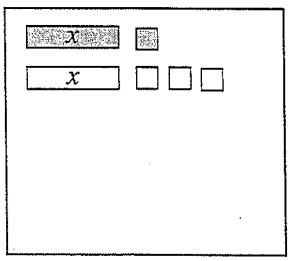
$<$ less than
$\leq$ less than or equal to
$>$ greater than
$\geq$ greater than or equal to

Review & Preview

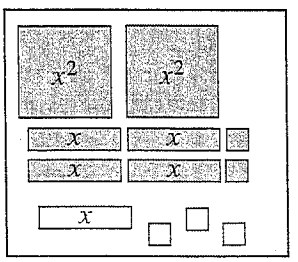
6-6. Write the expression shown on each of the Expression Mats below. Then simplify them by making zeros and combining like terms.



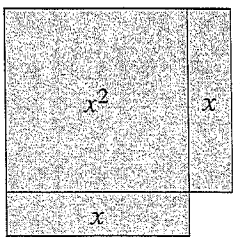
a.



b.



6-7. Which expressions are equivalent to the perimeter of the shape? How do you know?



a. $x+3+3x+1$

b. $2x+4+x$

c. $4x+4$

d. $2x+2+2x+2$

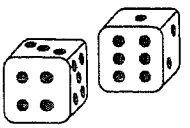
4

6-8. Simplify the following expressions.

a. $-\frac{3}{4}-\frac{2}{5}$ b. $\frac{7}{8}-\frac{2}{3}$ c. $\frac{1}{3}-\frac{5}{6}$

d. $1\frac{2}{3}+(-\frac{2}{5})$ e. $\frac{4}{7}-(-\frac{3}{8})$ f. $-4\frac{1}{2}+3\frac{1}{9}$

6-9. Desmond is rolling a standard six-sided number cube. He plans to roll it 72 times.



a. About how many times would you expect Desmond to roll a 4? Why?

b. About how many times would you expect him to roll an even number? Why?

c. Desmond kept track of his results for all 72 rolls. The table at right shows some of his results.

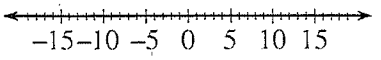
Based on his partial results, how many times did he roll a 5 or a 6?

Result	Number of Outcomes
1	9
2	14
3	11
4	8

6-10. In parts (a) through (c) below, you will see pairs of quantities. For each pair of quantities, use words to write a sentence that describes the relationship. For example, "\$5, \$8" could be, "\$8 is three more than \$5."

a. \$13, \$39 b. 25 feet, 17 feet c. 38 lbs., 19 lbs.

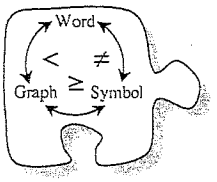
6-11. Copy each part below on your paper. Then use the number line to help you fill in < (less than) or > (greater than) on the blank line.



a. $-5 \_\_ -2$ b. $8 \_\_ -1$ c. $-5 \_\_ 0$ d. $-15 \_\_ -14$

5

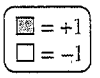
6.1.2 What if I cannot tell?



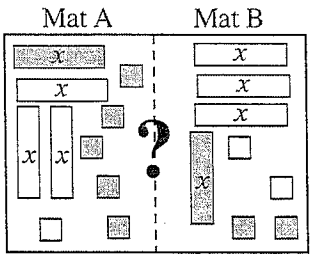
Comparing Quantities with Variables

Have you ever tried to make a decision when the information you have is uncertain? Perhaps you have tried to make plans on a summer day only to learn that it *might* rain. In that case, your decision might have been based on the weather, such as, "I will go swimming if it does not rain, or stay home and play video games if it does rain." Sometimes in mathematics, solutions might depend on something you do not know, like the value of the variable. Today you will study this kind of situation.

6-12. For each of the problems below, build the given expressions on your Expression Comparison Mat. Then use the simplification strategies of removing zeros and simplifying by removing matching pairs of tiles to determine which side is greater, if possible. Record your steps on the Lesson 6.1.2 Resource Page.



Steps ← a.

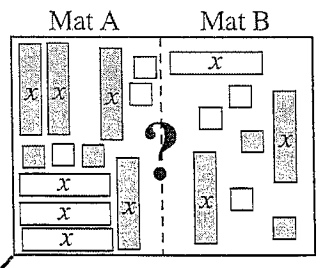


b. Mat A: $2(x+3)-4$

Mat B: $3x+(-1)-x+4$

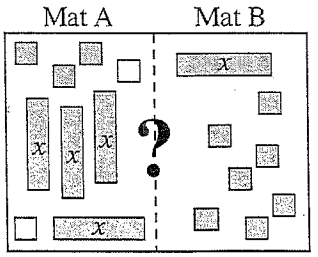
Steps

c.



Steps

d.

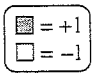


Steps

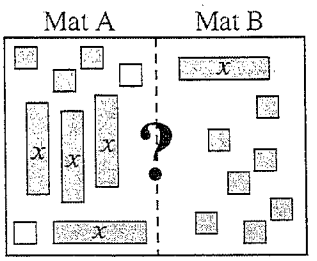
6

6-13. WHAT HAPPENED?

When Ignacio and Oliver compared the expressions in part (d) of problem 6-12, they could not figure out which side was greater.



a. Is it always possible to determine which side of the Expression Comparison Mat is greater (has the greater value)? Why or why not? Be prepared to share your reasoning.

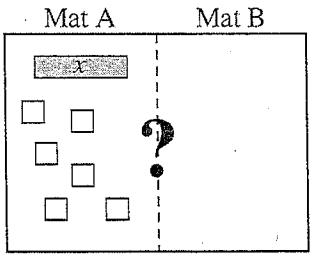


b. How is it possible for Mat A to have the greater value?

c. How is it possible for Mat B to have the greater value?

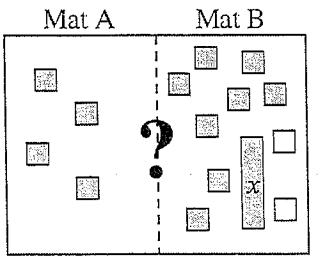
d. In what other way can Mat A and B be related? Explain.

6-14. Ignacio and Oliver are playing another game with the algebra tiles. After they simplify two new expressions, they are left with the expressions on their mats shown at right. They could not tell which part of the mat is greater just by looking.



a. One way to compare the mats is to separate the x-tiles and the unit tiles on different sides of the mat. Work with your team to find a way to have only x-tiles on Mat A. Make sure that you are able to justify that your moves are legal.

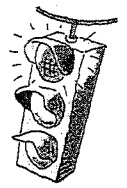
b. Using the same reasoning from part (a), what would you do to have only the variable on Mat B in the Expression Comparison Mat at right?



c. Write a short note to Ignacio and Oliver explaining this new strategy. Feel free to give it a name so it is easier for them to remember.

7

6-15. Ignacio and Oliver are trying to decide if there are other ways to change expressions on the Expression Comparison Mat without affecting which side is greater. They have invented some new strategies and described them below.



Your Task: For each of the moves below:

- Build the Expression Comparison Mats on your paper.
- Follow each set of directions for the mat shown in each strategy below.
- Determine if the move in the strategy is valid for maintaining the relationship between the two expressions. Be prepared to justify your response.

Strategy #1

Valid Move?

"If you have a mat like the one drawn below, you can add the same number of tiles to both sides. In this case, I added 3 negative tiles to both sides."

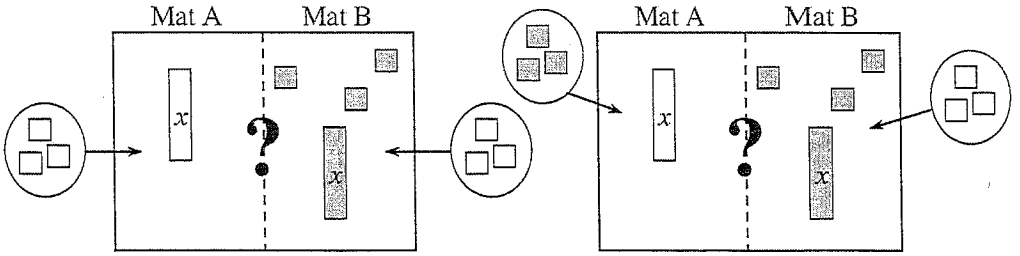
Strategy #2

Valid move?

"On a mat like the one below, I added +3 to Mat A and added –3 to Mat B."

= +1
= –1

Why?

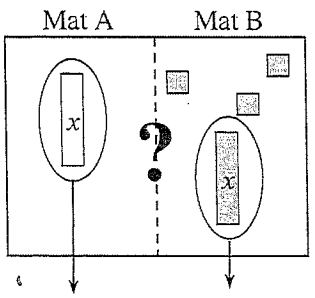


Why?

Strategy #3

Valid Move?

"To simplify, I removed a positive x-tile from one side and a negative x-tile from the other side."

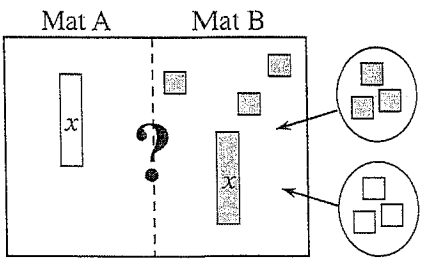


Why?

Strategy #4

"On a mat like the one below, I would add three zero pairs to Mat B."

Valid Move?



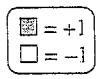
Why?

6-16. LEARNING LOG

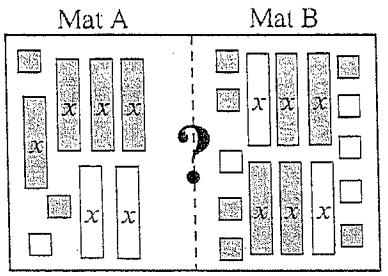
Summarize the methods you learned about Simplifying Expressions (Legal Moves) from the previous two lessons.

Review & Preview

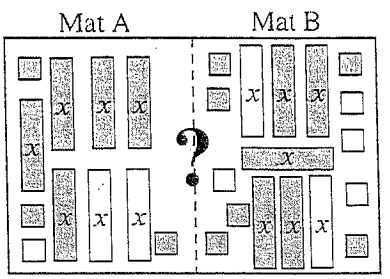
6-17. Write an algebraic expression for each mat below. Then use the legal moves that you have developed to simplify each mat. If possible, decide which expression is greater.



a.



b.



6-18. When solving a problem about the perimeter of a rectangle using the 5-D Process, Herman built the expression below.

$$\text{Perimeter} = x + x + 4x + 4x \text{ feet}$$

a. Draw a rectangle and label its sides based on Herman's expression.

b. What is the relationship between the base and height of Herman's rectangle? How can you tell?

c. If the perimeter of the rectangle is 60 feet, how long are the base and height of Herman's rectangle? Show how you know.

6-19. Evaluate the expressions below.

a. $5^2 \cdot (-3) - 4 \cdot 6 + 7$

b. $-3 \cdot (6 + 4 \cdot 2)$

c. $9 + 8 \div (-4) - 12$

d. $2^3 - 3 \cdot 4 + 6(-1 + 2)$

e. $4 + (3 + 4)^2$

f. $\frac{8-13}{10}$

9

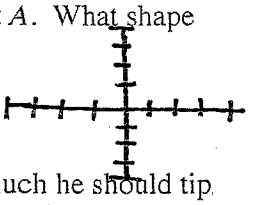
6-20. Write each of the following expressions in two ways, one with parentheses and one without. For example, $4(x-3)$ can be written $4x-12$.

a. A number reduced by 3 and then multiplied by 2.

b. A number increased by 7 and then multiplied by 5.

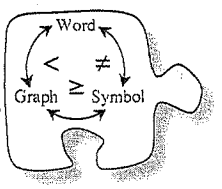
c. Ten times a number, and then add twenty.

6-21. Graph these points on a coordinate grid: $A(-2,0)$, $B(0,4)$, $C(4,1)$, $D(2,-3)$. Connect the points in order, with point D connected to point A. What shape have you created?



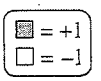
6-22. Alan was paying a dinner check, but he was not sure how much he should tip for his bill of \$27.38. If a 15% tip is standard, about how much should Alan leave for the server?

6.1.3 Where do the solutions begin and end?



One Variable Inequalities

You have used Expression Comparison Mats to compare two expressions and have found that sometimes it is possible to determine which expression is greater. In this lesson, you will again compare expressions. This time, you will find the values for the variable that make one expression greater than the other.



6-23. Maria has been recording her work to see which side of an Expression Comparison Mat is greater, but she has been called away. Garth looked at her work, but he cannot figure out what Maria did to get from one step to another.

Mat A	Mat B
$5x+2+(-6)$	$2x+2+(-8)$
$5x+(-4)$	$2x+(-6)$
$3x+(-4)$	-6
$3x$	-2

Look at Maria's work above and help Garth by building the expressions on your mat and simplifying them. Write him a note explaining what Maria did to get from one step to another.

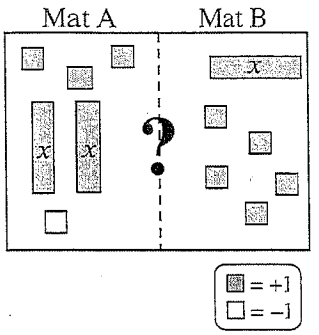
10

6-24. Compare the expressions $2 + 2x + (-3)$ on Mat A and $2x + (-4) + 1$ on Mat B using algebra tiles. Use Maria's method of recording to show your steps. Make sure you record each step so that your teacher or others could see what you did on your Expression Comparison Mat.

a. Which mat is greater?

b. Use symbols such as $<$, $=$, or $>$ to show the relationship between the final expressions on Mat A and Mat B.

6-25. Maria and Garth were playing a game with the algebra tiles. They each grabbed a handful of tiles and put them on the Expression Comparison Mat at right to see whose side had greater value.



Maria said, *"I have Mat A and my side has more value."* Garth, who had Mat B, disagreed with her.

a. Write expressions for Mat A and Mat B.

b. Work with your team to simplify the expressions on the Expression Comparison Mat while carefully recording your work for each step on your paper with symbols. Can you tell whose side is greater? Why or why not?

c. With your team, find at least four values for x that would make the expression on Maria's side (Mat A) greater than the expression on Garth's side (Mat B). Be prepared to share your values with the class.

d. Any value for x that makes Mat A greater than Mat B is a solution to the inequality $2x+3+(-1)>x+5$. This is read, *"Two x plus three plus negative one is greater than x plus five."*

Share your solutions with another team and see if you have the same solutions as the other team does.

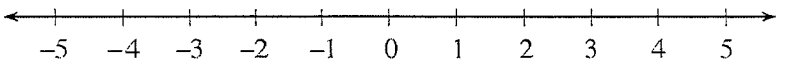
6-26. Karla had a hard time keeping track of all of the solutions to the inequality in problem 6-25 in her head. She decided to try to organize her answers. First she needed to know more about the problem.

a. Is there a greatest number that is a solution? Discuss this question with your team and be prepared to share your ideas with the class.

b. Is there a smallest number that is a solution? Again, be prepared to share your team's thinking with the class.

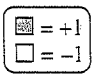
c. What is special about the point where the solutions end? (This number is called the **boundary point**.) In other words, what relationship does this number have to the two expressions being compared?

d. Karla was tired of listing so many solutions and wanted a quick way to show all of the solutions to this inequality. She decided to draw a number line like the one below.



On your own paper, draw a number line such as the one above then follow your teacher's directions to represent the answer to this question on your number line.

6-27. Now consider the inequality $2x+5<3$, which can be read *"Two x plus five is less than 3."*



Build the inequality on your Expression Comparison Mat and record each step on your paper using symbols. Work with your team to describe the least and greatest solutions to the inequality and draw your solution on a number line. Be prepared to justify your ideas.

6-28. Jerry and Ken were solving the inequality $6>2x+2$. They set up the inequality on their Expression Comparison Mat and simplified it.

a. Write a sentence in words to represent the original inequality.

b. What did they get on each side of the mat when they simplified? Record your work on your paper.

c. Graph all the solutions to this inequality on a number line.

READ OUT LOUD!

MATH NOTES

METHODS AND MEANINGS

Algebra Vocabulary

Variable: A letter or symbol that represents one or more numbers.

Expression: A combination of numbers, variables, and operation symbols. An expression does not contain an equal sign. For example, $2x + 3(5 - 2x) + 8$. Also, $5 - 2x$ is a smaller expression within the larger expression.

Term: Parts of the expression separated by addition and subtraction. For example, in the expression $2x + 3(5 - 2x) + 8$, the three terms are $2x$, $3(5 - 2x)$, and 8. The expression $5 - 2x$ has two terms, 5 and $2x$.

Coefficient: The numerical part of a term. In the expression $2x + 3(5 - 2x) + 8$, 2 is the coefficient of $2x$. In the expression $7x - 15x^2$, both 7 and 15 are coefficients.

Constant term: A number that is not multiplied by a variable. In the example above, 8 is a constant term. The number 3 is not a constant term because it is multiplied by a variable inside the parentheses.

Factor: Part of a multiplication expression. In the expression $3(5 - 2x)$, 3 and $5 - 2x$ are factors

Review & Preview

6-29. Graph each of the following inequalities on a number line.

a. $x > 3$ b. $x \leq 5$ c. $x \geq -4$

6-30. Write an algebraic expression for each situation. For example, 5 less than a number can be expressed as $n - 5$.

a. 7 more than a number b. Twice a number

6-31. MATH TALK – Use a separate sheet of paper for more space

Read the Math Notes box in this lesson to review commonly used algebra vocabulary. Then consider the expression below as you answer the following questions.

$$3x^2+7-2(4x+1)$$

a. Name the constant term.

b. What are the two factors in $2(4x+1)$? What are the two factors in $4x$?

c. Write an expression with a variable m, a coefficient -3, and a constant of 17.

d. Use the words coefficient, constant term, term, expression, and variable, to discuss $4x^2+11y-37$.

e. Use the words factor, product, quotient, and sum to describe the parts of $\frac{5-m}{n}-2-8(m+n)$.

6-32. Hector has a part-time job at a garage. He gets a paycheck of $820 every four weeks.

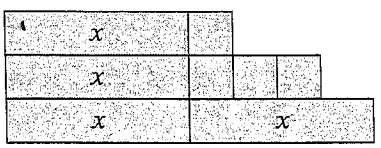
a. Hector has to pay 15% of his income in taxes. How much money does he pay in taxes each paycheck? Show your thinking with a diagram and calculations.

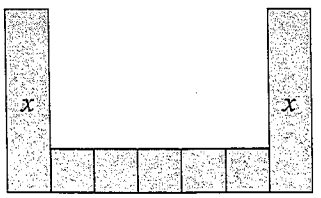
b. Hector took a 1-week vacation, so his next paycheck will only be for 3 weeks of work. What percentage of his regular pay should he expect to receive? How much is that?

c. The garage owner is impressed with Hector's work and is giving him a 10% raise. How much will Hector be paid when he receives his next 4-week paycheck?

6-33. A fair number cube labeled 1, 2, 3, 4, 5, and 6 is rolled 100 times. About how many times would you expect the number 3 to appear?

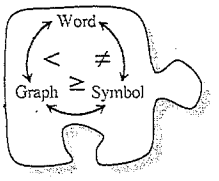
6-34. Find the perimeter and area of each algebra tile shape below. Be sure to combine like terms.

a. 

b. 

14

6.1.4 How can I find all solutions?



Solving One Variable Inequalities

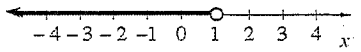
In this lesson, you will work with your team to develop and describe a process for solving linear inequalities. As you work, use the following questions to focus your discussion.

What is a solution?

What do all of the solutions have in common?

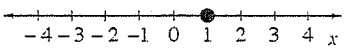
What is the greatest solution? What is the smallest solution?

6-35. Jerry and Ken were working on solving the inequality $3x - 1 \leq 2x$. They found the boundary point and Ken made the number line graph shown at right.



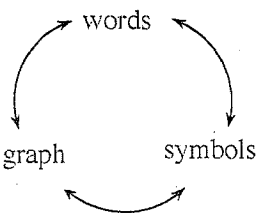
Jerry noticed a problem. *"Doesn't the line at the bottom of the ≤ symbol mean that it includes the equal part? That means that $x = 1$ is also a solution. How could we show that?"*

"Hmmm," Jerry said. *"Well, the solution $x = 1$ would look like this on a number line. Is there a way that we can combine the two number lines?"*



Discuss this idea with your team and be prepared to share your ideas with the class.

6-36. The diagram at right shows three possible ways to represent inequality statements. Review the meanings of the inequality symbols $>, <, \geq$, and $\leq$ with your team. Then, generate the two missing representations from each inequality described in parts (a) through (c) below.

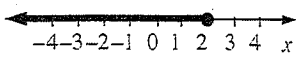


Graph

a. $x < -1\frac{1}{2}$ → words

b. x is greater than or equal to two.

symbols → graph

words ← c. 

→ symbols

6-37. WHEN IS THE BOUNDARY POINT INCLUDED?

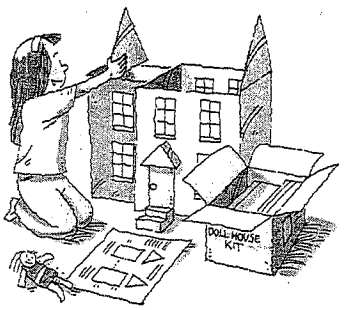
Represent the solution for each of the variables described below as an inequality on a number line and with symbols. show both

a. The speed limit on certain freeways is 65 miles per hour. Let x represent any speed that could get a speeding ticket.

b. You brought \$10 to the mall. Let y represent any amount of money you can spend.

c. To ride your favorite roller coaster, you must be at least five feet tall but less than seven feet tall. Let h represent any height that can ride the roller coaster.

6-38. Ellie was still working on her dollhouse. She has boards that are two different lengths. One long board is 54 inches.



a. The length of the short board is unknown. Ellie put three short boards end-to-end and then added her 12-inch ruler end-to-end. The total length was still less than the 54-inch board. Draw a picture showing how the short and long boards are related.

b. Write an inequality that represents the relationship between the short boards and 54 inches shown in your diagram in part (a). Be sure to state what your variable represents.

c. What are possible lengths of the short board? Show your answer as an inequality and on a number line.

STOP
V-in

6-39. Jordyn, Teri, and Morgan are going to have a kite-flying contest. Jordyn and Teri each have one roll of kite string. They also each have 45 yards of extra string. Morgan has three rolls of kite string plus 10 yards of extra string. All of the rolls of string are the same length. The girls want to see who can fly their kite the highest.

a. Since Jordyn and Teri have fewer rolls of kite string, they decide to tie their string together so their kite can fly higher. Write at least two expressions to show how much kite string Jordyn and Teri have. Let x represent the number of yards of string on one roll.

b. Write an expression to show how much kite string Morgan has. Again, let x be the number of yards of string on one roll.

c. How long does a roll of string have to be for Jordyn and Teri to be able to fly their kite higher than Morgan's kite? Show your answer as an inequality and on a number line.

d. How long does a roll of string have to be for Morgan to be able to fly her kite higher than Jordyn and Teri's kite? Show your answer as an inequality and on a number line.

e. What length would the roll of string have to be for the girls' kites to fly at the same height?

6-40. **Additional Challenge:** Travis loves trains! Today he is beginning a train ride from Madison, Wisconsin all the way to Seattle, Washington.

Try This!

Shortly after the train left the station in Madison, Travis fell asleep. When he woke up, it was dark outside and he had no idea how long he had been asleep. A fellow passenger told him they had already passed La Crosse, which is 135 miles from Madison. If the train travels at an average speed of 50 miles per hour, at least how long has Travis been asleep? Represent this problem with an inequality and then solve it.

6-41. LEARNING LOG

Work with your team to describe each step of your process for finding boundary points and deciding what part of the number line to shade. Then write down each step in your learning log. Be sure to illustrate your ideas with examples. Title this entry "Finding Boundary Points" and label it with today's date.

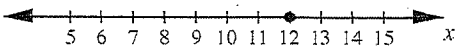
METHODS AND MEANINGS

Graphing Inequalities

MATH NOTES

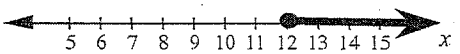
To solve and graph an inequality with one variable, first treat the problem as if it were an equality and solve the problem. The solution to the equality is called the **boundary point**. For example, to solve $x-4\geq 8$, first solve $x-4=8$. The solution $x=12$ is the boundary point for the inequality $x-4\geq 8$.

Since the original inequality is true when $x=12$, place your boundary point on the number line as a solid point. Then test one value on either side in the *original* inequality by substituting it into the original inequality. This will determine which set of numbers makes the inequality true. Write the inequality solution and extend an arrow onto the number line in the direction of the side that makes the inequality true. This is shown with the examples of $x=8$ and $x=15$ above. Therefore, the solution is $x\geq 12$ (also shown on the number line).

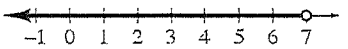


Test: $x=8$
$(8)-4\geq -8$
$4\geq -8$
FALSE!

Test: $x=15$
$(15)-4\geq 8$
$11\geq -8$
TRUE!



When the inequality is $<$ or $>$, the boundary point is *not* included in the answer. On a number line, this would be indicated with an open circle at the boundary point. For example, the graph of $x<7$ is shown below.



Review & Preview

6-42. Solve each of the following inequalities. Represent the solutions algebraically (with symbols) and graphically (on a number line).

a. $3x-3<2-2x$

b. $\frac{4}{5}x\geq 8$

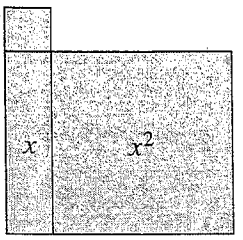
6-43. Determine whether each of the numbers below is a solution to the inequality $3x-2<2-2x$. Show all of your work.

a. 2 b. $\frac{1}{2}$ c. -3 d. $\frac{2}{3}$

6-44. Evaluate the expressions below using $x=-2$, $y=-5$, and $z=3$.

a. xyz b. $3(x+y)$ c. $\frac{z+2}{y}+1$

6-45. ~~On your paper, sketch the algebra tile shape at right.~~ Write an expression for the perimeter, and then find the perimeter for each of the given values of x.



a. $x=7$ cm b. $x=5.5$ cm

6-46. Alden found a partially completed 5-D table:

	Define			**Do**	**Decide** Target 74
Trial 1:	15	$2(15)=30$	$15+2=17$	$15+30+17=$	62 too small
Trial 2:	18	$2(18)=36$	$18+2=20$	$18+36+20=$	74 just right

a. Create a word problem that could have been solved using this table.

b. What words would you put above the numbers in the three empty sections in the "Trial" and "Define" parts of the table?

c. What word(s) would you put above the "Do" column?

6-47. Beth is filling a small backyard pool with a garden hose. The pool holds 30 gallons of water. After 5 minutes, the pool is about one-fourth full.

a. Assuming that the water is flowing at a constant rate, about how much water is going into the pool each minute?

b. About how long will it take to fill the pool?

19

Expression Comparison Mat